# <span id="page-0-0"></span>Package: NBPSeq (via r-universe)

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Title Negative Binomial Models for RNA-Sequencing Data

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- Description Negative Binomial (NB) models for two-group comparisons and regression inferences from RNA-Sequencing Data.
- **Depends**  $R (= 3.0.0)$

Imports splines

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## **Contents**





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#### Description

Negative binomial (NB) two-group and regression models for RNA-Sequencing data analysis.

### Details

See the examples of [test.coefficient](#page-55-1) and [exact.nb.test](#page-16-1) for typical workflows of using this package.

arab *Arabidopsis RNA-Seq Data Set*

### Description

An RNA-Seq dataset from a pilot study of the defense response of Arabidopsis to infection by bacteria. We performed RNA-Seq experiments on three independent biological samples from each of the two treatment groups. The matrix contains the frequencies of RNA-Seq reads mapped to genes in a reference database. Rows correspond to genes and columns correspond to independent biological samples.

#### Usage

data(arab)

### Format

A 26222 by 6 matrix of RNA-Seq read frequencies.

#### <span id="page-3-0"></span>Details

We challenged leaves of Arabidopsis with the defense-eliciting ∆*hrcC* mutant of *Pseudomonas syringae* pathovar *tomato* DC3000. We also infiltrated leaves of Arabidopsis with 10mM MgCl2 as a mock inoculation. RNA was isolated 7 hours after inoculation, enriched for mRNA and prepared for RNA-Seq. We sequenced one replicate per channel on the Illumina Genome Analyzer (http://www.illumina.com). The length of the RNA-Seq reads can vary in length depending on user preference and the sequencing instrument. The dataset used here are derived from a 36-cycle sequencing reaction, that we trimmed to 25mers. We used an in-house computational pipeline to process, align, and assign RNA-Seq reads to genes according to a reference database we developed for Arabidopsis.

#### Author(s)

Jason S Cumbie <cumbiej@onid.orst.edu> and Jeff H Chang <changj@cgrb.oregonstate.edu>.

#### References

Di Y, Schafer DW, Cumbie JS, and Chang JH (2011): "The NBP Negative Binomial Model for Assessing Differential Gene Expression from RNA-Seq", Statistical Applications in Genetics and Molecular Biology, 10 (1).



#### Description

Compute the probability of observing values of  $(S1, S2)$  that are more extreme than  $(S1, S2)$  given that S1+S2=s1+s2 for a pair of Negative Binomial (NB) random variables (S1, S2) with mean and size parameters (mu1, kappa1) and (mu2, kappa2) respectively.

#### Usage

compute.tail.prob(s1, s2, mu1, mu2, kappa1, kappa2)

#### Arguments



### <span id="page-4-0"></span>disp.by.group 5

#### Details

This function computes the probabily of  $(S1, S2)$  for all values of S1 and S2 such that  $S1+S2=1+s2$ , then sums over the probabilites that are less than or equal to that of the observed values (s1, s2). In context of DE test using RNA-Seq data after thinning, S1 and S2 are often sums of iid NB random variables (and are thus NB random variables too).

The current implementation can be slow if  $s1 + s2$  is large.

Potential improvements: For computing the one-sided tail probability of  $Pr(S1 \leq s1 | S1 + S2 = s1 + s2)$ , there might be a faster way. The conditional distribution can be also approximated by saddlepoint methods. If S1 and S2 are sum of two subsets of iid random variables, the saddle point approximation would be very accurate.

#### Value

a number giving the probability of observing a (S1, S2) that is as or more extreme than (s1, s2) given that S1+S2=s1+s2.

disp.by.group *Specify a dispersion model where the parameters of the model will be estimated separately for different groups*

#### Description

Specify a dispersion model where the parameters of the model will be estimated separately for different groups

#### Usage

```
disp.by.group(disp.fun, grp.ids, predictor, subset,
 predictor.label = "Predictor", ...)
```
### Arguments

disp.fun grp.ids predictor subset predictor.label

...

#### Value

a list,

Specify a NBP dispersion model. The parameters of the specified model are to be estimated from the data using the function optim.disp.apl or optim.disp.pl.

#### Usage

```
disp.nbp(counts, eff.lib.sizes, x, phi.pre = 0.1, mu.lower = 1,
 mu.upper = Inf)
```
### Arguments



### Details

Under this NBP model, the log dispersion is modeled as a linear function of the preliminary estimates of the log mean realtive frequencies (pi.pre):

 $log(\text{phi}) = par[1] + par[2] * log(\text{pi.pre/pi.offset}),$ 

where pi.offset is 1e-4.

Under this parameterization, par[1] is the dispersion value when the estimated relative frequency is 1e-4 (or 100 RPM).

#### Value

a list



#### <span id="page-6-0"></span>disp.nbq 7



<span id="page-6-1"></span>disp.nbq *(private) Specify a NBQ dispersion model*



### Description

Specify a NBQ dispersion model. The unknown parameters in the specified model are to be estimated using the function optim.disp.pl or optim.disp.apl.

### Usage

disp.nbq(counts, eff.lib.sizes, x, phi.pre = 0.1, mu.lower = 1, mu.upper = Inf, pi.offset = median(pi.pre[subset, ]))

### Arguments



#### Details

Under this NBQ model, the dispersion is modeled as a quadratic function of the preliminary estimates of the log mean realtive frequencies (pi.pre):

 $log(\pi h i) = par[1] + par[2] * z + par[3] * z<sup>2</sup>,$ 

where z = log(pi.pre/pi.offset). By default, pi.offset is the median of pi.pre[subset,].

#### Value





<span id="page-7-0"></span>

<span id="page-7-1"></span>

disp.nbs *(private) Specify a NBS dispersion model*

### **Description**

Specify a NBS dispersion model. The specified model are to be estimated using the function optim.disp.pl or optim.disp.apl. Under this NBS model, the dispersion is modeled as a smooth function (a natural cubic spline function) of the preliminary estimates of the log mean realtive frequencies (pi.pre).

### Usage

```
disp.nbs(counts, eff.lib.sizes, x, df = 6, phi.pre = 0.1, mu.lower = 1,
 mu.upper = Inf)
```
### Arguments



#### Details

disp.nbs calls the function ns to generate a set of spline bases, using log(pi.pre) (converted to a vector) as the predictor variable. Linear combinations of these spline bases are smooth functions of log(pi.pre). The return value includes a function, fun, to be optimized by optim.disp.pl or optim.disp.apl. The parameter of that function is a vector of linear combination coefficients of the spline bases.

df+2 nodes are used when constructing the splint bases. The Boundary.nodes are placed at the min and max values of log(pi.pre). Two nodes are placed at the 0.05 and 0.95th quantiles of log(pi.pre) and an additional df-2 inner nodes are equally spaced between the two nodes.

It is a challenging issue to determine the optimal number and placement of the nodes.

### <span id="page-8-0"></span>Value

a list



disp.predictor.mu *Dispersion precitor*

### Description

Dispersion precitor

### Usage

```
disp.predictor.mu(nb.data, x, phi.pre = 0.1, mu.lower = 1, mu.upper = Inf)
```
### Arguments

nb.data x phi.pre mu.lower mu.upper

### Value

a logical vector

Specify a piecewise constant (step) dispersion model. The specified model are to be estimated using the function optim.disp.pl or optim.disp.apl.

### Usage

```
disp.step(counts, eff.lib.sizes, x, df = 1, knots = NULL, phi.pre = 0.1,
 mu.lower = 1, mu.upper = Inf)
```
### Arguments



### Details

Under this model, the dispersion is modeled as a step (piecewise constant) function.

### Value



<span id="page-10-0"></span>Dispersion Models *(private) Specify a NB2, NBP, NBS, NBS, or STEP dispersion model*

#### Description

(private) Specify a NB2, NBP, NBS, NBS, or STEP dispersion model

#### Usage

```
disp.fun.nb2(predictor, subset, offset = NULL,
 predictor.label = "Predictor", par.init = -1)
disp.fun.nbp(predictor, subset, offset = median(predictor[subset, ]),
 predictor.label = "Predictor", par.init = c(log(0.1), 0),
 par.lower = c(log(1e-20), -1.1), par.upper = c(0, 0.1))
disp.fun.nbq(predictor, subset, offset = median(predictor[subset, ]),
 predictor.label = "Predictor", par.init = c(log(0.1), 0, 0),
 par.lower = c(log(1e-20), -1, -0.2), par.upper = c(0, 1, 0.2))
disp.fun.nbs(predictor, subset, offset = NULL,
 predictor.label = "Predictor", df = 6, par.init = rep(-1, df))
disp.fun.step(predictor, subset, offset = NULL,
 predictor.label = "Predictor", df = 6, knots = NULL,
 par.init = rep(-1, df)
```
### Arguments



### Details

Specify a NBP dispersion model. The parameters of the specified model are to be estimated from the data using the function optim.disp.apl or optim.disp.pl.

Under the NBP model, the log dispersion is modeled as a linear function of specified predictor with a scalar offset,

 $log(\text{phi}) = \text{par}[1] + \text{par}[2] * \text{log}(\text{predictor/offset}).$ 

Under this parameterization, par[1] is the dispersion value when the value of predictor equals the offset. This function will return a function (and related settings) to be estimated by either optim.disp.apl or optim.disp.pl. The logical vector subset specifieds which rows will be used when estimating the paramters (par) of the dispersion model.

Once estimated, the dispersion function will be applied to all values of the predictor matrix. Care needs to be taken to either avoid NA/Inf values when preparing the predictor matrix or handle NA/Inf values afterwards (e.g., when performing hypothesis tests).

#### Value

a list



<span id="page-11-1"></span>

### Description

Fit a parametric dispersion model to RNA-Seq counts data prepared by [prepare.nbp](#page-48-1). The model parameters are estimated from the pseudo counts: thinned/down-sampled counts that have the same effective library size.

#### Usage

```
estimate.disp(obj, model = "NBQ", print.level = 1, ...)
```
#### Arguments



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#### <span id="page-12-0"></span>estimate.disp 13

#### Details

For each individual gene i, a negative binomial (NB) distribution uses a dispersion parameter  $\phi_i$  to capture the extra-Poisson variation between biological replicates: the NB model imposes a meanvariance relationship  $\sigma_i^2 = \mu_i + \phi_i \mu_i^2$ . In many RNA-Seq data sets, the dispersion parameter  $\phi_i$ tends to vary with the mean  $\mu_i$ . We proposed to capture the dispersion-mean dependence using parametric models.

With this function, estimate.disp, users can choose from three parametric models: NB2, NBP and NBQ (default).

Under the NB2 model, the dispersion parameter is a constant and does not vary with the mean expression levels.

Under the NBP model, the log dispersion is modeled as a linear function of preliminarily estimated log mean relative frequencies (pi.pre):

 $log(\text{phi}) = \text{par}[1] + \text{par}[2] * \text{log}(\text{pi}.\text{pref}(\text{pi}.\text{offset}),$ 

Under the NBQ model, the log dispersion is modeled as a quadratic function of preliminarily estimated log mean relative frequencies (pi.pre):

 $log(phi) = par[1] + par[2] * log(pi.pre/pi.offset) + par[3] * (log(pi.pre/pi.offset))^2;$ 

The NBQ model is more flexible than the NBP and NB2 models, and is the current default option.

In the NBP and NBQ models, pi.offset is fixed to be 1e-4, so par[1] corresponds to the dispersion level when the relative mean frequency is 100 reads per million (RPM).

The dispersion parameters are estimated from the pseudo counts (counts adjusted to have the same effective library sizes). The parameters are estimated by maximizing the log conditional likelihood of the model parameters given the row sums. The log conditional likelihood is computed for each gene in each treatment group and then summed over genes and treatment groups.

### Value

The list obj from the input with some added components summarizing the fitted dispersion model. Users can print and plot the output to see brief summaries of the fitted dispersion model. The output is otherwise not intended for use by end users directly.

#### Note

Users should call prepare, nbp before calling this function. The function prepare, nbp will normalize the counts and adjust the counts so that the effective library sizes are approximately the same (computing the conditional likelihood requires the library sizes to be the same).

#### References

Di Y, Schafer DW, Cumbie JS, and Chang JH (2011): "The NBP Negative Binomial Model for Assessing Differential Gene Expression from RNA-Seq", Statistical Applications in Genetics and Molecular Biology, 10 (1).

#### See Also

[nbp.test](#page-37-1), [exact.nb.test](#page-16-1)

#### <span id="page-13-0"></span>Examples

## See the example for nb.exact.test

<span id="page-13-1"></span>estimate.dispersion *Estimate Negative Binomial Dispersion*

#### Description

Estimate NB dispersion by modeling it as a parametric function of preliminarily estimated log mean relative frequencies.

#### Usage

```
estimate.dispersion(nb.data, x, model = "NBQ", predictor = "pi",
 method = "MAPL", fast = TRUE, ...)
```
#### Arguments



### Details

We use a negative binomial (NB) distribution to model the read frequency of gene i in sample j. A negative binomial (NB) distribution uses a dispersion parameter  $\phi_{ij}$  to model the extra-Poisson variation between biological replicates. Under the NB model, the mean-variance relationship of a single read count satisfies  $\sigma_{ij}^2 = \mu_{ij} + \phi_{ij}\mu_{ij}^2$ . Due to the typically small sample sizes of RNA-Seq experiments, estimating the NB dispersion  $\phi_{ij}$  for each gene i separately is not reliable. One can pool information across genes and biological samples by modeling  $\phi_{ij}$  as a function of the mean frequencies and library sizes.

Under the NB2 model, the dispersion is a constant across all genes and samples.

Under the NBP model, the log dispersion is modeled as a linear function of the preliminary estimates of the log mean relative frequencies (pi.pre):

 $log(\text{phi}) = \text{par}[1] + \text{par}[2] * \text{log}(\text{pi}.\text{pref}(\text{pi}.\text{offset}),$ 

where pi.offset is 1e-4.

Under the NBQ model, the dispersion is modeled as a quadratic function of the preliminary estimates of the log mean relative frequencies (pi.pre):

<span id="page-14-0"></span> $log(\text{phi}) = \text{par}[1] + \text{par}[2] * z + \text{par}[3] * z^2$ ,

where  $z = log(pi.pre/pi.offset)$ . By default, pi.offset is the median of pi.pre[subset,].

Under this NBS model, the dispersion is modeled as a smooth function (a natural cubic spline function) of the preliminary estimates of the log mean relative frequencies (pi.pre).

Under the "step" model, the dispersion is modeled as a step (piecewise constant) function.

### Value

a list with following components:



### Note

Currently, it is unclear whether a dispersion-modeling approach will outperform a more basic approach where regression model is fitted to each gene separately without considering the dispersionmean dependence. Clarifying the power-robustness of the dispersion-modeling approach is an ongoing research topic.

#### Examples

## See the example for test.coefficient.

<span id="page-14-1"></span>estimate.norm.factors *Estiamte Normalization Factors*

### **Description**

estimate.norm.factors estiamtes normalization factors to account for apparent reduction or increase in relative frequencies of non-differentially expressing genes as a result of compensating the increased or decreased relative frequencies of truly differentially expressing genes.

#### Usage

```
estimate.norm.factors(counts, lib.sizes = colSums(counts),
 method = "AH2010")
```
#### Arguments



#### Details

We take gene expression to be indicated by relative frequency of RNA-Seq reads mapped to a gene, relative to library sizes (column sums of the count matrix). Since the relative frequencies sum to 1 in each library (one column of the count matrix), the increased relative frequencies of truly over expressed genes in each column must be accompanied by decreased relative frequencies of other genes, even when those others do not truly differentially express. If not accounted for, this may give a false impression of biological relevance (see, e.g., Robinson and Oshlack (2010), for some examples.) A simple fix is to compute the relative frequencies relative to effective library sizes library sizes multiplied by normalization factors.

#### Value

a vector of normalization factors.

#### References

Anders, S. and W. Huber (2010): "Differential expression analysis for sequence count data," Genome Biol., 11, R106.

Robinson, M. D. and A. Oshlack (2010): "A scaling normalization method for differential expression analysis of RNA-seq data," Genome Biol., 11, R25.

#### Examples

```
## Load Arabidopsis data
data(arab)
```

```
## Estimate normalization factors using the method of Anders and Huber (2010)
norm.factors = estimate.norm.factors(arab);
print(norm.factors);
```
<span id="page-16-1"></span><span id="page-16-0"></span>

exact.nb.test performs the Robinson and Smyth exact negative binomial (NB) test for differential gene expression on each gene and summarizes the results using p-values and q-values (FDR).

#### Usage

```
exact.nb.test(obj, grp1, grp2, print.level = 1)
```
#### Arguments



#### Details

The negative binomial (NB) distribution offers a more realistic model for RNA-Seq count variability and still permits an exact (non-asymptotic) test for comparing expression levels in two groups.

For each gene, let  $S_1$ ,  $S_2$  be the sums of gene counts from all biological replicates in each group. The exact NB test is based on the conditional distribution of  $S_1|S_1+S_2$ : a value of  $S_1$  that is too big or too small, relative to the sum  $S_1 + S_2$ , indicates evidence for differential gene expression. When the effective library sizes are the same in all replicates and the dispersion parameters are known, we can determine the probability functions of  $S_1$ ,  $S_2$  explicitly. The exact p-value is computed as the total conditional probability of all possible values of  $(S_1, S_2)$  that have the same sum as but are more extreme than the observed values of  $(S_1, S_2)$ .

Note that we assume that the NB dispersion parameters for the two groups are the same and library sizes (column totals of the count matrix) are the same.

#### Value

the list obj from the input with the following added components:



<span id="page-17-0"></span>

#### Note

Before calling [exact.nb.test](#page-16-1), the user should call [estimate.norm.factors](#page-14-1) to estimate normalization factors, call [prepare.nbp](#page-48-1) to adjust library sizes, and call [estimate.disp](#page-11-1) to fit a dispersion model. The exact NB test will be performed using pseudo.counts in the list obj, which are normalized and adjusted to have the same effective library sizes (column sums of the count matrix, multiplied by normalization factors).

Users not interested in fine tuning the underlying statistical model should use [nbp.test](#page-37-1) instead. The all-in-one function [nbp.test](#page-37-1) uses sensible approaches to normalize the counts, estimate the NBP model parameters and test for differential gene expression.

A test will be performed on a row (a gene) only when the total row count is nonzero, otherwise an NA value will be assigned to the corresponding p-value and q-value.

#### See Also

[nbp.test](#page-37-1).

### Examples

```
## Load Arabidopsis data
data(arab);
## Specify treatment groups
## grp.ids = c(1, 1, 1, 2, 2, 2); # Numbers or strings are both OK
grp.ids = rep(c("mock", "hrec"), each=3);## Estimate normalization factors
norm.factors = estimate.norm.factors(arab);
print(norm.factors);
## Prepare an NBP object, adjust the library sizes by thinning the
## counts. For demonstration purpose, only use the first 100 rows of
## the arab data.
set.seed(999);
obj = prepare.nbp(arab[1:100,], grp.ids, lib.size=colSums(arab), norm.factors=norm.factors);
print(obj);
## Fit a dispersion model (NBQ by default)
obj = estimate.disp(obj);
plot(obj);
```
## Perform exact NB test

### <span id="page-18-0"></span>filter.mu.pre 19

```
## grp1 = 1;
## grp2 = 2;
grp1 = "mock";grp2 = "hrec";obj = exact.nb.test(obj, grp1, grp2);
## Print and plot results
print(obj);
par(mfrow=c(3,2));
plot(obj);
```


### Description

Create a logical vector specifyfing the subset of rows to be used when estimating the dispersion model

### Usage

```
filter.mu.pre(nb.data, x, mu.lower = 1, mu.upper = Inf, phi.pre = 0.1)
```
### Arguments

nb x mu.lower

mu.upper

### Value

a logical vector specifyfing the subset of rows to be used when estimating the dispersion model

<span id="page-19-0"></span>fit.nb.glm.1 *Fit a single negative binomial (NB) log-linear regression model with known dispersion paramreters*

### Description

Fit a NB log-linear regression model: find the MLE of the regression coefficients and compute likelihood of the fitted model, the score vector, and the Fisher and observed information.

#### Usage

fit.nb.glm.1(y, s, x, phi, beta $0 = \text{rep}(NA, \dim(x)[2]), \ldots)$ 

#### Arguments



### Details

Under the NB regression model, the components of y follow a NB distribution with means  $mu = s$ exp(x' beta) and dispersion parameters phi.

The function will call [irls.nb.1](#page-25-1) to find MLE of the regression coefficients (which uses the iteratively reweighted least squres (ILRS) algorithm).

#### Value



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### Note

The information matries, i and j, will be computed for all all components of beta—including known components.

fit.nb.glm.1u *Fit a single negative binomial (NB) log-linear regression model with a common unknown dispersion paramreter*

### Description

Fit a single negative binomial (NB) log-linear regression model with a common unknown dispersion paramreter.

#### Usage

fit.nb.glm.1u(y, s, x, phi = NA, beta $\theta$  = rep(NA, dim(x)[2]),  $kappa = 1/\phi$ hi, info.kappa = TRUE, ...)

#### Arguments



### Details

Find the MLE of the dipsersion parameter and the regression coefficients in a NB regression model.

Under the NB regression model, the components of y follow a NB distribution with means  $mu = s$ exp(x' beta) and a common dispersion parameter phi.

### <span id="page-21-0"></span>Value

a list



### Note

When the disperison is known, the user should specify only one of phi or kappa. Whenever phi is specified (non-NA), kappa will be set to 1/phi.

The observed information matrix, j, will be computed for all parameters—kappa and all components of beta (including known components). It will be computed at the estimated values of (phi, beta) or (kappa, beta), which can be unconstrained or constrained MLEs depending on how the arguments phi (or kappa) and beta are specified.

TODO: allow computing the information matrix using phi or log(kappa) as parameter

<span id="page-21-1"></span>

### Description

(private) Extract row means of the pseudo counts for the specified group from an nbp object.

### Usage

```
get.mean.hat(obj, grp.id)
```
#### Arguments



<span id="page-22-0"></span>

(private) Retrieve nbp parameters for one of the treatment groups from an nbp object

### Usage

get.nbp.pars(obj, grp.id)

### Arguments



### Value





### Description

(private) Extract row relative means of the pseudo counts for the specified group from an nbp object.

### Usage

get.rel.mean(obj, grp.id)

### Arguments



<span id="page-23-0"></span>

(private) Extract estimated variance from the oupput of nbp-mcle or nbp-test

### Usage

get.var.hat(obj, grp.id)

### Arguments





hist2d *2-d Histogram*

### Description

Commpute a 2-d histogram of the given data values. (not implemented yet: If plot == TRUE, plot the resulting histogram.)

#### Usage

hist2d(x, y, xlim = range(x), ylim = range(y), nbins)

#### Arguments



### Details

This funciton divides the xlim x ylim region into nbins x nbins equal-sized cells and count the number of (x,y) points in each cell.

#### <span id="page-24-0"></span>hoa.1d 25

### Value



The list can be passed to image() directly for potting.

### Note

Only points inside the region defined by xlim x ylim (inclusive) will be counted. For each cell, the lower boundaries are closed and upper boundaries are open. A small number will be added to the upper limits in xlim and ylim so that no points will be on the region's upper boundaries.



### Description

(private) One-dimensional HOA test for a regression coefficient in an NB GLM model

### Usage

hoa.1d(y, s, x, phi, beta0, tol.mu =  $0.001/length(y)$ , alternative = "two.sided", print.level = 1)

### Arguments



#### Value

test statistics and p-values of HOA, LR, and Wald tests

<span id="page-25-0"></span>

(private) HOA test for regression coefficients in an NBP GLM model

#### Usage

```
hoa.hd(y, s, x, phi, beta0, tol.mu = 0.001/length(y), print.level = 1)
```
### Arguments



#### Value

test statistics and p-values of HOA, LR, and Wald tests

```
irls.nb (private) Estiamte the regression coefficients in an NB GLM model
```
### Description

Estimate the regression coefficients in an NBP GLM model for each gene

### Usage

```
irls.nb(y, s, x, phi, beta0 = rep(NA, ncol(x)), mustart = NULL, ...,print<math>level = 0
```
#### <span id="page-26-0"></span>irls.nb.1 27

### Arguments



### Value

beta a K vector, the MLE of the regression coefficients.

irls.nb.1 *Estimate the regression coefficients in an NB GLM model*

### Description

Estimate the regression coefficients in an NB GLM model with known dispersion parameters

### Usage

```
irls.nb.1(y, s, x, phi, beta\theta = rep(NA, p), mustart = NULL, maxit = 50,
  tol.mu = 0.001/length(y), print.level = 0)
```
### Arguments



### <span id="page-27-0"></span>Details

This function estimates the regression coefficients using iterative reweighted least squares (IRLS) algorithm, which is equivalent to Fisher scoring. The implementation is based on glm.fit.

Users can choose to fix some regression coefficients by specifying beta0. (This is useful when fitting a model under a null hypothesis.)

#### Value

a list of the following components:



### l.nb *(private) The Log Likelihood of a NB Model*

### **Description**

The log likelihood of the NB model under the mean shape parameterization

#### Usage

l.nb(kappa, mu, y)

### Arguments



### Details

This function call dnbinom to compute the log likelihood from each data point and sum the results over all data points. kappa, mu and y should have compatible dimensions.

#### Value

the log likelihood of the NB model parameterized by (kappa, mu)

<span id="page-28-0"></span>

Specify a NB2 dispersion model. The parameter of the specified model are to be estimated from the data using the function optim.pcl.

#### Usage

```
## S3 method for class 'phi.nb2'
log(par, pi)
```
### Arguments

par a number, log dispersion

### Details

Under this NB2 model, the log dispersion is a constant.

### Value

a vector of length m, log dipserison.

log.phi.nbp *(private) A NBP dispersion model*

#### Description

Specify a NBP dispersion model. The parameters of the specified model are to be estimated from the data using the function optim.pcl.

### Usage

```
## S3 method for class 'phi.nbp'
log(par, pi, pi.offset = 1e-04)
```
### Arguments



### <span id="page-29-0"></span>Details

Under this NBP model, the log dispersion is modeled as a linear function of the log mean realtive frequencies (pi.pre):

 $log(\text{phi}) = \text{par}[1] + \text{par}[2] * \text{log}(\text{pi}.\text{pre/pi}.\text{offset}),$ 

where the default value of pi.offset is 1e-4.

### Value

a vector of length m, log dipserison.

log.phi.nbq *(private) A NBQ dispersion model*

### Description

Specify a NBQ dispersion model. The parameters of the specified model are to be estimated from the data using the function optim.pcl.

#### Usage

## S3 method for class 'phi.nbq'  $log(par, pi, pi.offset = 1e-04)$ 

### Arguments



### Details

Under this NBQ model, the log dispersion is modeled as a quadratic function of the log mean realtive frequencies (pi):

 $log(\text{phi}) = \text{par}[1] + \text{par}[2] * \text{log}(\text{pi/pi} \cdot \text{offset}) + \text{par}[3] * (\text{log}(\text{pi/pi} \cdot \text{offset}))$ <sup>2</sup>;

where the (default) value of pi.offset is 1e-4.

### Value

a vector of length m, log dipserison.

<span id="page-30-0"></span>

Plot log (base 2) fold change vs average expression in RPM (two-group pooled) (i.e., an MA plot) and highlight differentially expressed genes on the plot.

#### Usage

```
ma.plot(test.out, top = NULL, q.cutoff = NULL, p.cutoff = NULL,
 col.sig = "magenta", main = "MA Plot", ...)
```
#### Arguments



### Details

Differentially expressed genes are those with smallest DE test p-values. The user has three options to specify the set of DE genes: the user can specify 1) the number of top genes to be declared as significant; 2) a q-value cutoff; or 3) a p-value cutoff.

The plot is based on the thinned counts. The units on the x-axis is RPM (reads per million mapped reads). We use RPM so that the results are more comparable between experiments with different sequencing depth (and thus different column totals in the count matrix). We exclude rows (genes) with 0 total counts after thinning.

#### Value

a vector, indices of top genes.

<span id="page-31-0"></span>

Specify a dispersion model. The parameters of the specified model are to be estimated from the data using the function optim.disp.apl or optim.disp.pl.

#### Usage

```
make.disp(nb.data, x, model, predictor, subset = filter.mu.pre(nb.data, x),
  ...)
```
### Arguments



#### Details

This functions calls disp. fun. < model> to specify a dispersion model (a list), using output from a call to disp.predictor.<predictor> as argument list, where <model> is model from the input in lower case (one of "nb2", "nbp", "nbq", "nbs" or "step") and <predictor> is predictor from the input (one of "pi", "mu", or "rs")

### Value

a list, output from the call to the funtion disp.fun.<model>.

<span id="page-32-1"></span><span id="page-32-0"></span>

Overlay an estimated mean-variance line on an existing mean-variance plot

### Usage

mv.line(mu, v, ...)

#### Arguments



### Details

Users should call mv.plot before calling this function.

If the length of theinput vectors (mu, v) is greater than 1000, then we will only use a subset of the input vectors.

mv.line.fitted *(private) Overlay an estimated mean-variance line*

### Description

Overlay an estimated mean-variance line on existing plot

#### Usage

```
mv.line.fitted(obj, ...)
```
### Arguments



### Details

This functions is a wrapper of  $mv$ . line. It takes a list (rather than two vectors) as input.

<span id="page-33-0"></span>

Users should call mv.plot before calling this function.

### See Also

[mv.line](#page-32-1)

mv.line.nbp *(private) Overlay a NBP mean-variance line on an existing plot*

### Description

Overlay an estimated mean-variance line on existing plot

#### Usage

mv.line.nbp(nbp.obj, grp.id, ...)

#### Arguments



### Details

This function extracts the estimated means and variances from an nbp object and then call mv. line to draw the mean-variance line on an existing plot

#### Note

Users should call mv.plot before calling this function.

### See Also

[prepare.nbp](#page-48-1), [nbp.test](#page-37-1), [mv.line](#page-32-1)

<span id="page-34-0"></span>

Mean-variance plot.

### Usage

```
mv.plot(counts, xlab = "mean", ylab = "variance",
  main = "variance vs mean", log = "xy", ...
```
### Arguments



#### Details

Rows with mean 0 or variance 0 will not be plotted.

mv.points *(private) Highlight a subset of points on the mean-variance plot*

### Description

Highlight a subset of points on the mean-variance plot

### Usage

```
mv.points(counts, subset, ...)
```
### Arguments



<span id="page-35-1"></span><span id="page-35-0"></span>

For each row of the input data matrix, nb.glm.test fits an NB log-linear regression model and performs large-sample tests for a one-dimensional regression coefficient.

#### Usage

```
nb.glm.test(counts, x, beta0, lib.sizes = colSums(counts),
 normalization.method = "AH2010", dispersion.model = "NBQ",
  tests = c("HOA", "LR", "Wald"), alternative = "two.sided",
  subset = 1:dim(counts)[1])
```
### Arguments



#### Details

nbp.glm.test provides a simple, one-stop interface to performing a series of core tasks in regression analysis of RNA-Seq data: it calls [estimate.norm.factors](#page-14-1) to estimate normalization factors;

#### <span id="page-36-0"></span>nb.glm.test 37

it calls [prepare.nb.data](#page-47-1) to create an NB data structure; it calls [estimate.dispersion](#page-13-1) to estimate the NB dispersion; and it calls [test.coefficient](#page-55-1) to test the regression coefficient.

To keep the interface simple, nbp.glm.test provides limited options for fine tuning models/parameters in each individual step. For more control over individual steps, advanced users can call [estimate.norm.factors](#page-14-1), [prepare.nb.data](#page-47-1), [estimate.dispersion](#page-13-1), and [test.coefficient](#page-55-1) directly, or even substitute one or more of them with their own versions.

### Value

A list containing the following components:



#### Examples

```
## Load Arabidopsis data
data(arab);
## Specify treatment structure
grp.ids = as.factor(c(1, 1, 1, 2, 2, 2));x = model.matrix(\neg grp.ids);## Specify the null hypothesis
## The null hypothesis is beta[1]=0 (beta[1] is the log fold change).
beta = c(NA, \emptyset);## Fit NB regression model and perform large sample tests.
## The step can take long if the number of genes is large
fit = nb.glm.test(arab, x, beta0, subset=1:50);## The result contains the data, the dispersion estimates and the test results
print(str(fit));
## Show HOA test results for top ten genes
subset = order(fit$test.results$HOA$p.values)[1:10];
cbind(fit$data$counts[subset,], fit$test.results$HOA[subset,]);
## Show LR test results
```

```
subset = order(fit$test.results$LR$p.values)[1:10];
cbind(fit$data$counts[subset,], fit$test.results$LR[subset,]);
```
<span id="page-37-1"></span><span id="page-37-0"></span>

nbp.test fits an NBP model to the RNA-Seq counts and performs Robinson and Smyth's exact NB test on each gene to assess differential gene expression between two groups.

#### Usage

```
nbp.test(counts, grp.ids, grp1, grp2, norm.factors = rep(1, dim(counts)[2]),
 model.disp = "NBQ", lib.sizes = colSums(counts), print.level = 1, ...)
```
#### Arguments



#### Details

nbp.test calls [prepare.nbp](#page-48-1) to create the NBP data structure, perform optional normalization and adjust library sizes, calls [estimate.disp](#page-11-1) to estimate the NBP dispersion parameters and [exact.nb.test](#page-16-1) to perform the exact NB test for differential gene expression on each gene. The results are summarized using p-values and q-values (FDR).

Overview: For assessing evidence for differential gene expression from RNA-Seq read counts, it is critical to adequately model the count variability between independent biological replicates. Negative binomial (NB) distribution offers a more realistic model for RNA-Seq count variability than Poisson distribution and still permits an exact (non-asymptotic) test for comparing two groups.

For each individual gene, an NB distribution uses a dispersion parameter  $\phi_i$  to model the extra-Poisson variation between biological replicates. Across all genes, parameter  $\phi_i$  tends to vary with the mean  $\mu_i$ . We capture the dispersion-mean dependence using a parametric model: NB2, NBP and NBQ. (See [estimate.disp](#page-11-1) for more details.)

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Count Normalization: We take gene expression to be indicated by relative frequency of RNA-Seq reads mapped to a gene, relative to library sizes (column sums of the count matrix). Since the relative frequencies sum to 1 in each library (one column of the count matrix), the increased relative frequencies of truly over expressed genes in each column must be accompanied by decreased relative frequencies of other genes, even when those others do not truly differentially express. Robinson and Oshlack (2010) presented examples where this problem is noticeable.

A simple fix is to compute the relative frequencies relative to effective library sizes—library sizes multiplied by normalization factors. By default, nbp.test assumes the normalization factors are 1 (i.e. no normalization is needed). Users can specify normalization factors through the argument norm.factors. Many authors (Robinson and Oshlack (2010), Anders and Huber (2010)) propose to estimate the normalization factors based on the assumption that most genes are NOT differentially expressed.

Library Size Adjustment: The exact test requires that the effective library sizes (column sums) of the count matrix multiplied by normalization factors) are approximately equal. By default, nbp.test will thin (downsample) the counts to make the effective library sizes equal. Thinning may lose statistical efficiency, but is unlikely to introduce bias.

#### Value

a list with the following components:



Due to thinning (random downsampling of counts), two identical calls to nbp.test may yield slightly different results. A random number seed can be used to make the results reproducible. The regression analysis method implemented in [nb.glm.test](#page-35-1) does not require thinning and can also be used to compare expression in two groups.

Advanced users can call [estimate.norm.factors](#page-14-1), [prepare.nbp](#page-48-1), [estimate.disp](#page-11-1), [exact.nb.test](#page-16-1) directly to have more control over modeling and testing.

### References

Di, Y, D. W. Schafer, J. S. Cumbie, and J. H. Chang (2011): "The NBP Negative Binomial Model for Assessing Differential Gene Expression from RNA-Seq", Statistical Applications in Genetics and Molecular Biology, 10 (1).

Robinson, M. D. and G. K. Smyth (2007): "Moderated statistical tests for assessing differences in tag abundance," Bioinformatics, 23, 2881-2887.

Robinson, M. D. and G. K. Smyth (2008): "Small-sample estimation of negative binomial dispersion, with applications to SAGE data," Biostatistics, 9, 321-332.

Anders, S. and W. Huber (2010): "Differential expression analysis for sequence count data," Genome Biol., 11, R106.

Robinson, M. D. and A. Oshlack (2010): "A scaling normalization method for differential expression analysis of RNA-seq data," Genome Biol., 11, R25.

#### See Also

[prepare.nbp](#page-48-1), [estimate.disp](#page-11-1), [exact.nb.test](#page-16-1).

#### Examples

```
## Load Arabidopsis data
data(arab);
## Specify treatment groups and ids of the two groups to be compared
grp.ids = c(1, 1, 1, 2, 2, 2);
grp1 = 1;grp2 = 2;## Estimate normalization factors
norm.factors = estimate.norm.factors(arab);
## Set a random number seed to make results reproducible
set.seed(999);
## Fit the NBP model and perform exact NB test for differential gene expression.
## For demonstration purpose, we will use the first 100 rows of the arab data.
res = nbp.test(arab[1:100,], grp.ids, grp1, grp2,
 lib.sizes = colSums(arab), norm.factors = norm.factors, print.level=3);
## The argument lib.sizes is needed since we only use a subset of
```
## rows. If all rows are used, the following will be adequate:

<span id="page-39-0"></span>

### **Note**

```
nll.log.phi.fun 41
```

```
##
## res = nbp.test(arab, grp.ids, grp1, grp2, norm.factors = norm.factors);
## Show top ten most differentially expressed genes
subset = order(res$p.values)[1:10];
print(res, subset);
## Count the number of differentially expressed genes (e.g. qvalue < 0.05)
alpha = 0.05;
sig.res = res$q.values < alpha;
table(sig.res);
## Show boxplots, MA-plot, mean-variance plot and mean-dispersion plot
par(mfrow=c(3,2));
plot(res);
```
nll.log.phi.fun *Negative profile conditional likelihood of the dispersion model*

### Description

Negative profile conditional likelihood of the dispersion model

#### Usage

```
nll.log.phi.fun(par, log.phi.fun, y, ls, n.grps, grps, grp.sizes, mu.lower,
 mu.upper, print.level)
```
### Arguments



#### Value

negative log likelihood of the dispersion function

<span id="page-41-0"></span>

Estimate the parameters in a dispersion model.

### Usage

```
optim.disp.apl(disp, counts, eff.lib.sizes, x, method = "L-BFGS-B",
 mustart = NULL, fast = FALSE, print<math>. level = 1, ...
```
### Arguments



### Details

The function will call the R funciton optim to mimimize the negative log adjusted profile likelihood of the dipserison model.

### Value

a list with components:

optim.disp.pl *(private) Estimate the parameters in a dispersion model*

### Description

Estimate the parameters in a dispersion model.

#### Usage

```
optim.disp.pl(disp, counts, eff.lib.sizes, x, method = "L-BFGS-B",
 mustart = NULL, fast = FALSE, ...)
```
#### <span id="page-42-0"></span>phi.line **43**

### Arguments



### Details

The function will call the R funciton optim to mimimize the negative log likelihood of the dipserison model.

### Value

a list with components:

<span id="page-42-1"></span>phi.line *(private) Overlay an mean-dispersion line on an esimtated plot*

### **Description**

Users should call vmr.plot before calling this function.

### Usage

phi.line(mu,  $v$ , alpha = 2, ...)

#### Arguments



### Details

If the length of theinput vectors (mu, v) is greater than 1000, then we will only use a subset of the input vectors.

The dispersion is computed from the mean mu and the variance v, using  $\phi = (v - \mu)/\mu^{\alpha}lph\alpha$ , where alpha=2 by default.

### <span id="page-43-0"></span>Note

Currently, we discards genes giving 0 mean or negative dispersion estimate (which can happen if sample variance is smaller than the sample mean).

phi.line.fitted *(private) Overlay an estimated mean-dispersion line on an existing plot*

### Description

Overlay an estimated mean-dispersion line on an existing plot

### Usage

```
phi.line.fitted(obj, alpha = 2, ...)
```
### Arguments



#### Details

This function is a wrapper of [phi.line](#page-42-1). It takes a list (rather than two separate vectors) as input.

### Note

Users should call phi.plot before calling this function.

#### See Also

[phi.line](#page-42-1)

<span id="page-44-0"></span>

Overlay an estimated mean-dispersion line on an existing plot

#### Usage

```
phi.line.nbp(nbp.obj, grp.id, alpha = 2, ...)
```
### Arguments



#### Details

This function extracts the estimated means and variances from an nbp object and then call phi. line to draw the mean-dispersion curve

#### Note

Users should call phi.plot before calling this function.

#### See Also

[prepare.nbp](#page-48-1), [nbp.test](#page-37-1), [phi.line](#page-42-1)



phi.plot *Plot estimated genewise NB2 dispersion parameter versus estimated mean*

### Description

Plot estimated NB2 dispersion parameter versus estimated mean

#### Usage

```
phi.plot(counts, alpha = 2, xlab = "mean", ylab = "phi.hat",
 main = "phi.hat vs mean", \log = "xy", ...)
```
### Arguments



### Details

phi.plot estimate the NB2 dispersion parameter for each gene separately by  $\phi = (v - \mu)/\mu^{\alpha}lpha$ , where  $\mu$  and  $v$  are sample mean and sample variance. By default,  $alpha = 2$ .

### Note

Currently, we discards genes giving 0 mean or negative dispersion estimate (which can happen if sample variance is smaller than the sample mean).



### Description

Boxplot and scatterplot matrix of relative frequencies (after normalization)

#### Usage

## S3 method for class 'nb.data'  $plot(x, resolution = 50, hlim = 0.25, clip = 128,$  $eps = 0.01, ...$ 

### Arguments



<span id="page-45-0"></span>

<span id="page-46-0"></span>plot.nb.dispersion *Plot the estimated dispersion as a function of the preliminarily estimated mean relative frequencies*

### Description

Plot the estimated dispersion as a function of the preliminarily estimated mean relative frequencies

#### Usage

## S3 method for class 'nb.dispersion'  $plot(x, \ldots)$ 

#### Arguments



plot.nbp *Diagnostic Plots for an NBP Object*

#### Description

For output from [nbp.test](#page-37-1), produce a boxplot, an MA plot, mean-variance plots (one for each group being compared), and mean-dispersion plots (one for each group being compared). On the mean-variance and the mean-dispersion plots, overlay curves corresponding to the estimated NBP model.

#### Usage

## S3 method for class 'nbp' plot(x, ...)

#### Arguments



#### See Also

[nbp.test](#page-37-1)

### Examples

## See the example for nbp.test

<span id="page-47-1"></span><span id="page-47-0"></span>

Create a data structure to hold the RNA-Seq read counts and other relevant information.

#### Usage

```
prepare.nb.data(counts, lib.sizes = colSums(counts),
 norm.factors = estimate.norm.factors(counts), tags = NULL)
```
### Arguments



### Value

A list containing the following components:



<span id="page-48-1"></span><span id="page-48-0"></span>prepare.nbp *Prepare the Data Structure for Exact NB test for Two-Group Comparison*

#### **Description**

Create the NBP data structure, (optionally) normalize the counts, and thin the counts to make the effective library sizes equal.

#### Usage

```
prepare.nbp(counts, grp.ids, lib.sizes = colSums(counts),
 norm.factors = NULL, thinning = TRUE, print.level = 1)
```
#### Arguments



### Details

#### Normalization

We take gene expression to be indicated by relative frequency of RNA-Seq reads mapped to a gene, relative to library sizes (column sums of the count matrix). Since the relative frequencies sum to 1 in each library (one column of the count matrix), the increased relative frequencies of truly over expressed genes in each column must be accompanied by decreased relative frequencies of other genes, even when those others do not truly differently express. Robinson and Oshlack (2010) presented examples where this problem is noticeable.

A simple fix is to compute the relative frequencies relative to effective library sizes—library sizes multiplied by normalization factors. Many authors (Robinson and Oshlack (2010), Anders and Huber (2010)) propose to estimate the normalization factors based on the assumption that most genes are NOT differentially expressed.

By default, prepare.nbp does not estimate the normalization factors, but can incorporate user specified normalization factors through the argument norm.factors.

Library Size Adjustment

The exact test requires that the effective library sizes (column sums of the count matrix multiplied by normalization factors) are approximately equal. By default, prepare.nbp will thin (downsample) the counts to make the effective library sizes equal. Thinning may lose statistical efficiency, but is unlikely to introduce bias.

#### Value

A list containing the following components:



#### Note

Due to thinning (random downsampling of counts), two identical calls to prepare.nbp may yield slightly different results. A random number seed can be used to make the results reproducible.

### See Also

[nbp.test](#page-37-1)

### Examples

## See the example for exact.nb.test

print.nb.data *Print summary of the nb counts*

### Description

Print summary of the nb counts

#### Usage

## S3 method for class 'nb.data'  $print(x, \ldots)$ 

<span id="page-49-0"></span>

### <span id="page-50-0"></span>Arguments



print.nb.dispersion *Print the estimated dispersion model*

### Description

Print the estimated dispersion model

### Usage

```
## S3 method for class 'nb.dispersion'
print(x, \ldots)
```
### Arguments





### Description

We simply print out the structure of x. (Currenlty the method is equivalent to  $print(str(x))$ .)

### Usage

```
## S3 method for class 'nb.test'
print(x, \ldots)
```
### Arguments



<span id="page-51-0"></span>

Print contents of an NBP object, output from [prepare.nbp](#page-48-1), [estimate.disp](#page-11-1), or [nbp.test](#page-37-1).

#### Usage

```
## S3 method for class 'nbp'
print(x, subset = 1:10, ...)
```
### Arguments



#### See Also

[nbp.test](#page-37-1).

#### Examples

## See the example for nbp.test

smart.plot.new *(private) An alternative to plot.default() for plotting a large number of densely distributed points.*

### Description

An alternative to plot.default() for plotting a large number of densely distributed points. This function can produce a visually almost identical plot using only a subset of the points. This is particular useful for reducing output file size when plots are written to eps files.

#### Usage

```
smart.plot.new(x, y = NULL, xlim = NULL, ylim = NULL, xlab = NULL,ylab = NULL, log = "", resolution = 50, col = gray((224:0)/256),
  clip = NULL, col.\text{clipped} = rgb(log2(1:256)/log2(256), 0, 0), ...
```
### smart.plot.new 53

#### Arguments



### Details

Writing plots with a large number of points to eps files can result in big files and lead to very slow rendering time.

Usually for a large number of points, a lot of them will overlap with each other. Plotting only a subset of selected non-overlapping points can give visually almost identical plots. Further more, the plots can be enhanced if using gray levels (the default setting) that are proportional to the number points overlapping with each plotted point.

This function scans the points sequentially. For each unmarked point that will be plotted, all points that overlap with it will be marked and not to plotted, and the number of overlapping points will be recorded. This is essentially producing a 2d histogram. The freqs of the points will be converted to gray levels, darker colors correspond to higher freqs.

### Value

(if plot=FALSE) a list



<span id="page-53-0"></span>smart.plot.old *(private) An alternative to plot.default() for plotting a large number of densely distributed points.*

### **Description**

An alternative to plot.default() for plotting a large number of densely distributed points. This function can produce a visually almost identical plot using only a subset of the points. This is particular useful for reducing output file size when plots are written to eps files.

#### Usage

```
smart.plot.old(x, y = NULL, xlim = NULL, ylim = NULL, xlaw = NULL,ylab = NULL, log = "", resolution = 100, plot = TRUE, col = NULL,
 clip = Inf, color.clipped = TRUE, ...)
```
#### Arguments



### Details

Writing plots with a large number of points to eps files can result in big files and lead to very slow rendering time.

Usually for a large number of points, a lot of them will overlap with each other. Plotting only a subset of selected non-overlapping points can give visually almost identical plots. Further more, the plots can be enhanced if using gray levels (the default setting) that are proportional to the number points overlapping with each plotted point.

This function scans the points sequentially. For each unmarked point that will be plotted, all points that overlap with it will be marked and not to plotted, and the number of overlapping points will be recorded. This is essentially producing a 2d histogram. The freqs of the points will be converted to gray levels, darker colors correspond to higher freqs.

### <span id="page-54-0"></span>smart.points 55

### Value

(if plot=FALSE) a list





## Description

See description of [smart.plot](#page-0-0) for more details.

### Usage

```
smart.points(x, y = NULL, resolution = 50, col = NULL, clip = Inf,color<u>c</u>ipped = TRUE, ...)
```
### Arguments



<span id="page-55-1"></span><span id="page-55-0"></span>

test.coefficient performs large-sample tests (higher-order asymptotic test, likelihood ratio test, and/or Wald test) for testing regression coefficients in an NB regression model.

#### Usage

```
test.coefficient(nb, dispersion, x, beta0, tests = c("HOA", "LR", "Wald"),
  alternative = "two.sided", subset = 1:m, print.level = 1)
```
### Arguments



#### Details

test.coefficient performs large-sample tests for a one-dimensional ( $q = 1$ ) component  $\psi$  of the p-dimensional regression coefficient  $\beta$ . The hypothesized value  $\psi_0$  of  $\psi$  is specified by the non-NA component of the vector beta0 in the input.

The likelihood ratio statistic,

 $\lambda = 2(l(\hat{\beta}) - l(\tilde{\beta})),$ 

converges in distribution to a chi-square distribution with 1 degree of freedom. The signed square root of the likelihood ratio statistic  $\lambda$ , also called the directed deviance,

$$
r = sign(\hat{\psi} - \psi_0)\sqrt{\lambda}
$$

converges to a standard normal distribution.

#### test.coefficient 57

For testing a one-dimensional parameter of interest, Barndorff-Nielsen (1986, 1991) showed that a modified directed

$$
r^* = r - \frac{1}{r} \log(z)
$$

is, in wide generality, asymptotically standard normally distributed to a higher order of accuracy than the directed deviance r itself, where  $z$  is an adjustment term. Tests based on high-order asymptotic adjustment to the likelihood ratio statistic, such as  $r^*$  or its approximation, are referred to as higher-order asymptotic (HOA) tests. They generally have better accuracy than corresponding unadjusted likelihood ratio tests, especially in situations where the sample size is small and/or when the number of nuisance parameters  $(p-q)$  is large. The implementation here is based on Skovgaard (2001). See Di et al. 2013 for more details.

#### Value

a list containing the following components:



### References

Barndorff-Nielsen, O. (1986): "Infereni on full or partial parameters based on the standardized signed log likelihood ratio," Biometrika, 73, 307-322

Barndorff-Nielsen, O. (1991): "Modified signed log likelihood ratio," Biometrika, 78, 557-563.

Skovgaard, I. (2001): "Likelihood asymptotics," Scandinavian Journal of Statistics, 28, 3-32.

Di Y, Schafer DW, Emerson SC, Chang JH (2013): "Higher order asymptotics for negative binomial regression inferences from RNA-sequencing data". Stat Appl Genet Mol Biol, 12(1), 49-70.

#### Examples

```
## Load Arabidopsis data
data(arab);
## Estimate normalization factors (we want to use the entire data set)
norm.factors = estimate.norm.factors(arab);
## Prepare the data
## For demonstration purpose, only the first 50 rows are used
nb.data = prepare.nb.data(arab[1:50,], lib.sizes = colSums(arab), norm.factors = norm.factors);
## For real analysis, we will use the entire data set, and can omit lib.sizes parameter)
## nb.data = prepare.nb.data(arab, norm.factors = norm.factors);
print(nb.data);
```
plot(nb.data);

```
## Specify the model matrix (experimental design)
grp.ids = as.factor(c(1, 1, 1, 2, 2, 2));x = model.matrix(\neg grp.ids);## Estimate dispersion model
dispersion = estimate.dispersion(nb.data, x);
print(dispersion);
plot(dispersion);
## Specify the null hypothesis
## The null hypothesis is beta[2]=0 (beta[2] is the log fold change).
beta = c(NA, 0);## Test regression coefficient
res = test.coefficient(nb.data, dispersion, x, beta0);
## The result contains the data, the dispersion estimates and the test results
print(str(res));
## Show HOA test results for top ten most differentially expressed genes
top = order(res$HOA$p.values)[1:10];
print(cbind(nb.data$counts[top,], res$HOA[top,]));
## Plot log fold change versus the fitted mean of sample 1 (analagous to an MA-plot).
plot(res$mu.tilde[,1], res$beta.hat[,2]/log(2), log="x",
     xlab="Fitted mean of sample 1 under the null",
     ylab="Log (base 2) fold change");
## Highlight top DE genes
points(res$mu.tilde[top,1], res$beta.hat[top,2]/log(2), col="magenta");
```


Thin (downsample) counts to make the effective library sizes equal.

#### Usage

```
thin.counts(y, current.lib.sizes = colSums(y),
  target.lib.sizes = min(current.lib.sizes))
```
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### Arguments

y an n by r matrix of counts current.lib.sizes an r vector indicating current estimated library sizes target.lib.sizes an r vector indicating target library sizes after thinning

### Details

The exact NB test for differential gene expression requires that the effective library sizes (column sums of the count matrix multiplied by normalization factors) are approximately equal. This function will thin (downsample) the counts to make the effective library sizes equal. Thinning may lose statistical efficiency, but is unlikely to introduce bias. The reason to use thinning, not scaling, is because Poisson counts after thinning are still Poisson, but Poisson counts after scaling will not be Poisson.

### Value

a list



[.nb.data *hello*

### Description

hello

#### Usage

## S3 method for class 'nb.data'  $x[i, j, ..., drop = FALSE]$ 

#### Arguments

x i j ... drop

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